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Junk Food, Health and Productivity: Taste, Price, Risk and Rationality

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Abstract

Junk-food consumption, health and productivity are analyzed within an expected-lifetime-utility-maximizing framework in which the probability of living and productivity rise with health and health deteriorate with the consumption of junk-food. So long that the junk food's relative taste-price differential is positive, the rational diet deviates from the physiologically optimal and renders the levels of health and productivity lower than the maximal. Taxing junk-food can eliminate this discrepancy but the outcome is not Pareto-superior. The value of health and the stationary junk-food consumption and health depend on the relative taste-price differential, survival and satisfaction elasticities and time preference-rate.

JEL classification: I12, D91

Keywords: junk-food, healthy-food, relative taste, relative price, health, risk, life-quality, productivity, rationality, self-control

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1. Introduction

Food is classified as junk or healthy in accordance with the concentration of ingredients whose presence in the human body beyond a critical level is harmful.[1] Significant quantities of these ingredients are contained in junk-food, whereas insignificant in healthy-food. Due to a high concentration of these ingredients, junk-food is tastier than its healthy substitute for some people. Due to cheaper ingredients and/or preparation process junk-food is often less expensive than the healthy substitute. These short-term taste and price aspects might be paramount for many people and hence generate a large deviation from the physiologically optimal diet as may be reflected by the prevalence of overweight and obesity and the existence of large fast-food and snack-food industries.[2] In the case of non-myopic people, the effects of short-term taste and price advantages of junk-food on its level of consumption are moderated by the risk stemming from the excessive consumption of its harmful ingredients.

This paper analyzes junk-food consumption and its implications for productivity and health from an economically rational perspective. It does so by incorporating the possible short-term taste and price advantages and the long-term risk disadvantage of junk-food vis-à-vis healthy-food into an expected-lifetime-utility-maximization framework. Consistently with Karen Dynan's (2000) empirical findings with panel household data, the present analysis assumes that food-consumption is neither addictive nor a formed habit. That is, the stocks of junk-food consumption and healthy-food consumption are not considered to moderate the individual's level of

[1] Examples of such ingredients are fat, sugar and salt.

[2] Trenton Smith (2004) analyzes the manipulative nature of advertisements in these industries. There are claims that the taste and price aspects have also been exploited by the least expected industry. Patricia Anderson and Kristin Butcher (2005) argue that, due to budgetary reasons, the availability of junk-foods in schools has been increased, and that the greater availability explains about one-fifth of the increase in average body mass index among adolescents in the United States over the last decade.

satisfaction from the flows of these commodities and hence are not included in the individual's utility function. The analysis focuses on the roles of price, taste and risk differences in explaining the rationally dietary composition. The rational food-consumers are assumed to have self-control and time-consistent preferences and to maximize their expected lifetime utility from consumption of junk-food and healthy-food subject to the evolution of their health and its implications for their productivity, prospects of survival (or, as alternatively interpreted, life-quality). Unlike Hugo Mialon and Sue Mialon (2005),[3] the junk-food's substitute is taken to be harmless: it contains insignificant quantities of harmful substances. Furthermore, its high price and/or low taste, as well as the higher level of activity of its healthier consumers, prevent excessive intake of calories.

In addition to reflecting on the composition of the individual's diet, the analysis highlights the effects of relative price, relative taste, food-consumption elasticity of utility, health-elasticity of survival, junk-food's health-erosion coefficient and time preferences on the long-run level of the individual's health and productivity. The model developed in section 2 displays the assumed relationships between satisfaction, health, productivity, budget, risk and the individual's diet. Section 3 presents the expected lifetime-utility from eating, its possible interpretation as life-quality-adjusted expected utility, the consumer's decision problem and the corner solutions of rational abstinence and rational indulgence. Section 4 discusses the properties of the composite diet and the value of health in the case of interior solution. Section 5 derives and analyzes the individual's stationary health in the interior-

[3] The substitute to the harmful good in Hugo Mialon and Sue Mialon's (2005) analysis is a less harmful good (e.g., light cigarettes and light beverages). The availability of a less harmful substitute does not necessarily improve the consumer's health. It reduces the risk for people with high taste for the more harmful good. However, it increases the risk for people with a sufficiently low taste for the more harmful good due to a large consumption of the less harmful good, which still contains significant quantities of harmful substances.

solution case. Section 6 highlights aggregate health and income aspects and identifies the individual and aggregate health and growth-maximizing-tax policy. Section 7 concludes.

2. Model

For simplicity, the model includes only two goods; healthy-food and junk-food; traded in perfectly competitive markets at time-invariant prices. The presentation of the assumed direct and indirect effects of the junk and healthy-food diet on the individual's satisfaction level, health, productivity, budget and risk employs the following definitions and notations:

j - a nucleus junk-food component of a meal;

h – a nucleus healthy-food component of a meal;

p - a positive scalar indicating a time-invariant price-ratio of the nucleus junk-food component and the nucleus health-food component (hereafter, relative price);

α - a positive scalar denoting the individual's taste-ratio of the nucleus junk-food component and the nucleus health-food component (hereafter, relative taste) and reflecting time-consistent tastes;

c_h^o - the physiologically optimal number of nucleus healthy-food components required for maintaining perfect health (i.e., the diet of a perfectly healthy person) – the physiologically optimal diet;

$c_h(t)$ - the individual's healthy-food consumption (i.e., the number of nucleus healthy-food components consumed) at t, $0 \le c_h(t) \le c_h{}^o$;

$c_j(t)$ - the individual's junk-food consumption (i.e., the number of nucleus junk-food components consumed) at t;

$c_j(t) + c_h(t)$ - the individual's diet at t;

$x(t)$ - the individual's health condition at t, a unit-interval index $0 \le x(t) \le 1$ with $x = 0$ representing terminal sickness and $x = 1$ perfect health;

$\hat{y}$ - a positive scalar indicating the individual's full-capacity income;

$y(t)$ - the individual's income at t ;

$\phi(t)$ - the probability density of dying at t;

$u(t)$ - the individual's utility from consumption at t;

ρ - a time-consistent personal rate of time preference, $0 < \rho < 1$; and

V - the individual's lifetime utility.

Productivity and income: Skill and employment opportunities determine the individual's full-capacity income $\hat{y}$. The individual's health determines the individual productivity — the extent to which the individual realizes her/his full-capacity income. Productivity reaches 1 when the individual is perfectly healthy and converges to 0 as the individual becomes terminally ill. Namely, the individual's instantaneous income is given by:

$$y(t) = x(t)\hat{y}\,. \tag{1}$$

Budget: Taking the price of healthy-food to be a numeraire, the individual's instantaneous budget constraint is:[4]

$$pc_j(t) + c_h(t) = x(t)\hat{y}\,. \tag{2}$$

[4] The presentation of the more general case of intertemporal-budget constraint with borrowing and lending requires the inclusion of an extra state variable (outstanding debt or credit) and interest rate. The consideration of such intertemporal budget constraint complicates the analysis tremendously while not being a major issue.

The right-hand side of the budget constraint reflects that the healthier the person the greater her/his spending on food. A possible explanation is that health is associated with a lower consumption of junk-food and a greater consumption of the usually more expensive healthy-food. Though not explicitly indicated, a spending increasing in health is also consistent with the casual observation that the healthier the person the more active she/he is and hence the greater her/his appetite and demand for food. Once reached, perfect health ($x = 1$) is maintained by adhering to the physiologically optimal diet ($c_j = 0$ and $c_h = c_h^o$). Correspondingly, and in recalling Eq. (1), the balanced-budget equation requires that the full-capacity income earned by a perfectly healthy person is equal to the cost of the physiologically optimal diet ($\hat{y} = c_h{}^o$). To let perfect health be achievable, this equality is assumed. With this assumption Eq. (2) can be rendered as

$$c_h(t) = x(t)c_h^o - pc_j(t)\,. \tag{3}$$

Instantaneous utility: Consistently with Karen Dynan's (2000) empirical findings of insignificant addiction, the instantaneous utility derived from consuming the two types of food is independent from past consumption. It is represented by a function $u(c_j(t), c_h(t))$ having the following properties. Food is essential: $u(0,0) = 0$. Yet neither junk-food nor healthy-food is essential by itself: $u(0, c_h), u(c_j, 0) > 0$. The marginal instantaneous satisfaction with respect to each type of food is positive and diminishing: $u_j, u_h > 0$, $u_{jj}, u_{hh} < 0$. Healthy-food and junk-food are substitutes: $u_{jh} < 0$. Consistently, the following explicit instantaneous-utility function is considered:

$$u_t = [\alpha c_j(t) + c_h(t)]^{\beta} \tag{4}$$

where $0 < \beta < 1$ is the elasticity of the individual's satisfaction from the composite diet.[5] Recalling Eq. (3), the instantaneous utility function can be further expressed as

$$u_t = [(\alpha - p)c_j(t) + x(t)c_h^o]^{\beta}\,. \tag{5}$$

Health: Health is deteriorated by eating junk-food and improved by a natural recovery process. The instantaneous change in the individual's health is represented by a logistic function displaying a diminishing relative health-improvement rate in junk-food consumption, a diminishing health-improvement rate (r) in the level of health, and a unit upper-bound and a zero lower-bound on the individual's health. Using $c_j(t)/c_h^o$ as an index of the excessive physiological inadequacy of the current diet vis-à-vis the currently affordable healthiest-diet, $c_h(t) = x(t)\hat{y}$, the evolution of the individual's health is presented by

$$\dot{x}(t) = \underbrace{\{1 - \delta[c_j(t)/c_h^o]\}[1 - x(t)]}_{r}x(t) \tag{6}$$

where, δ is a positive scalar indicating the marginal adverse effect of physiologically inadequate diet on the relative rate of improvement of the individual's health. When the individual refrains from consuming junk-food her/his current recovery rate $(\dot{x}(t)/x(t))$ is maximal and equal to the recovery rate $1 - x(t)$ facilitated by the currently affordable healthiest diet. The interpretation of the health-motion equation is enhanced by noting that

$$1 - \delta[c_j(t)/c_h^o] = [\dot{x}(t)/x(t)]/[1 - x(t)]\,. \tag{6'}$$

[5] The satisfaction-elasticities with respect to junk-food and healthy-food are not identical. Their ratio is equal to the product of the relative taste and quantities: $\alpha(c_j/c_h)$.

Namely, $1-\delta[c_j(t)/c_h^o]$ is the current rate of health-change ($\dot{x}(t)/x(t)$) relatively to the currently affordable maximal recovery rate ($1-x(t)$). This current *relative health-change rate* is hindered by the current junk-food consumption and is negative for $c_j(t) > c_h^o/\delta$.[6]

Risk: The probability of survival (living beyond *t*) rises with the individual's health. It is equal to one when the individual is perfectly healthy ($x=1$), converges to zero as the individual's health diminishes and is, for tractability, isoelastic. In formal terms, let $F(t)$ be the cumulative distribution function associated with the probability density of dying ($\phi(t)$) and, consequently, $\Phi(t)=1-F(t)$ be the probability of living beyond t, it is assumed that $\Phi(t)=\Phi(x(t))$ with $\Phi_x > 0$ (a positive health effect) $\lim_{x\to 1}\Phi = 1$ and $\lim_{x\to 0}\Phi = 0$. It is further assumed that the survival elasticity ($\Phi_x \frac{x}{\Phi}$) is equal to a positive scalar η. Namely,

$$\Phi(t) = x(t)^{\eta}. \tag{7}$$

Since $0 \le x \le 1$, $0 \le \Phi \le 1$ for any $\eta > 0$. Consequently, the rate of change of the survival probability is proportional to the rate of change of health: $\frac{\dot{\Phi}}{\Phi} = \eta \frac{\dot{x}}{x}$.

[6] The case of a negative relative health-improvement rate does not violate the assumption that x lies within the (positive) unit interval as long as the initial value of x is smaller than 1. Furthermore, when x is close to zero and the consumption of junk-food is lower than c_h^o/δ, $1-\delta[c_j/c_h^o]$ can be interpreted as the junk-food weakened recovery rate from a near-death situation. Had the healthy good contained significant quantities of harmful ingredients, the Mialon-Mialon proposition indicated in footnote 2 could be reproduced by specifying the health-motion equation as $\dot{x}(t) = \{1-\delta[c_j(t)+\mu c_h(t)]\}[1-x(t)]x(t)$, where $0<\mu<1$ indicates the harm caused by consuming a unit of healthy-food relatively to that caused by a unit of junk-food.

3. Lifetime-utility maximization and the cases of abstinence and indulgence

Rational individuals with self-control choose their diet path so as to maximize expected utility from consumption over the remainder of their life, subject to their health motion equation. Since the duration of life is random, they multiply their accumulated utility from food between the starting point of their planning horizon ($\int_0^t e^{-\rho\tau} u_\tau d\tau$) by their possible time of death t ($\phi(t)$). The products of $\phi(t)$ and $\int_0^t e^{-\rho\tau} u_\tau d\tau$ associated with any possible life expectancy $0 \le t \le \infty$ are considered by such consumers. The sum of all these products, $\int_0^\infty \phi(t) \int_0^t e^{-\rho\tau} u_\tau d\tau dt$, is these consumers' expected lifetime-utility which, by integrating by parts and recalling Eq. (7), can be expressed as:[7]

$$E(V) = \int_0^\infty \Phi(x(t)) e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t} x(t)^\eta u_t dt \,. \tag{8}$$

The right-hand-side term provides an alternative interpretation of *E*(*V*): one that is based on the association of quality of life and health. The number of quality-adjusted life-years is used in cost-benefit analysis of health investment projects as an index of well-being. It combines the duration of life and health condition into a single utility index. (Cf. Han Bleichrodt, 1995; and Han Bleichrodt and John Quiggin, 1999.) Likewise, $0 \le x(t)^\eta \le 1$ can be alternatively regarded as a life-quality index and $\int_0^\infty e^{-\rho t} x(t)^\eta u_t dt$ as the quality-adjusted lifetime-utility from food-consumption.

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[7] See Amnon Levy (2002a, 2002b) for proof and use of $\int_0^\infty \phi(t) \int_0^t e^{-\rho\tau} u_\tau d\tau dt = \int_0^\infty \Phi(t) e^{-\rho t} u_t dt$ for analyzing the prevalence of overweight and HIV-AIDS among rational people.

Substituting Eq. (5) into Eq. (8) for u_t, the rational junk-food consumption path is found by $\max_{\{c_j\}} \int_0^{\infty} e^{-\rho t} x(t)^{\eta} [(\alpha - p)c_j(t) + x(t)c_h^o]^{\beta} dt$ subject to the health-motion equation (6).

When analyzing junk-food consumption the polar phenomena of abstinence and indulgence deserve attention. Abstinence and indulgence are usually attributed to dogmatism and loss of self-control, respectively. In the context of the present optimal-control peoblem, abstinence and indulgence may arise as corner solutions.

PROPOSITION 1 (*Rational abstinence*): *If the relative price of junk-food exceeds the relative taste of junk-food ($p > \alpha$), a junk-free diet is rationally optimal, converging to the physiologically optimal diet, and maximizing health and productivity.*

PROPOSITION 2 *(Rational indulgence): If the relative taste of junk-food exceeds the relative price of junk-food ($\alpha > p$) and the individual is myopic ($\rho \to \infty$) a junk-full diet is rationally optimal but maximizing health and productivity loss and gradually leading to complete self-destruction.* (See Appendix A for proof.)

4. Composite diet and the value of health

The analysis of the rational choice of junk-healthy-food composition continues under the assumptions of positive relative taste-price differential ($\alpha - p > 0$) and non-myopia. The Hamiltonian corresponding to the aforementioned constrained maximization problem is:

$$H = e^{-\rho t} x^{\eta} [(\alpha - p)c_j + xc_h^o]^{\beta} + \lambda[1 - \delta(c_j / c_h^o)](1 - x)x \tag{9}$$

where the co-state variable λ indicates the shadow present value of the individual's health. (The time-index is omitted for tractability.) In addition to the state-equation (2), maximum expected lifetime satisfaction from eating requires that the change in the individual's valuation of her/his health is given by:

$$\dot{\lambda} = -[\eta x^{\eta-1} Z^{\beta} - x^{\eta} \beta Z^{\beta-1} c_h^o] e^{-\rho t} - \lambda(1-2x)(1-\delta c_j / c_h^o) \tag{10}$$

and that along the rational food-consumption path the marginal satisfaction from eating junk-food, discounted by both the individual's time preferences and prospects of survival, is equal the value of the marginal health-damage caused by eating junk-food:

$$x^{\eta} e^{-\rho t} \beta Z^{\beta-1}(\alpha - p) - \lambda(\delta / c_h^o) x(1-x) = 0 \tag{11}$$

where $Z \equiv (\alpha - p)c_j + xc_h^o$.[8]

PROPOSITION 3: *The value of health for a rational person with is increased (reduced) by eating junk-food when her/his health is better (worse) than a critical level, which rises with the ratio of the elasticity of survival with respect to health (η) to the elasticity of satisfaction with respect to eating (β) and is given by* $(1+\eta/\beta)/(2+\eta/\beta)$. (See Appendix A for proof.)

As explained in a greater detail in Appendix B, the instantaneous change in the rationally self-controlled junk-food consumption is given by:

[8] Since $0 < \beta < 1$ the Hamiltonian is concave in c_j. However, neither $e^{-\rho t} x^{\eta}[(\alpha - p)c_j + xc_h^o]^{\beta}$ nor $[1 - \delta(c_j / c_h^o)](1-x)x$ is necessarily concave in the state variable (x). In turn, the Hamiltonian is not necessarily concave in x. In such a case, the Mangasarian's theorem on the sufficiency of Pontryagin's maximum-principle conditions is not valid. Nonconcavity of a Hamiltonian in its state variable plays a crucial role in generating unstable steady states, and possibly, a Dechert-Nishimura-Skiba point.

$$\dot{c}_j = -\left\{\frac{(\alpha - p)c_j + xc_h^o}{(1-\beta)(\alpha - p)}\right\}\left\{\rho - \frac{(\frac{\eta}{\beta x}Z - \gamma\hat{y})(1-x)(\delta / c_h^o)x}{(\alpha - p)} - [\frac{\eta}{x} - \frac{(1-\beta)c_h^o}{Z}]\dot{x}\right\}. \qquad (12)$$

The system comprising Eq. (12) and Eq. (6) portrays the joint evolution of the rationally self-controlled junk-food consumption and health. The complexity of this system reflects that the effects of the model parameters on the transition of junk-food consumption and health are not clear. Interior steady states (SS) are analyzed in the following section for exploring the possible long-run levels of the rational junk-food consumption and health.

5. Stationary junk-food consumption and health

In steady state, the junk-food consumption is $c_{j_{ss}} = c_h^o / \delta$ and, as shown in Appendix B, the health and (in view of Eq. (1)) productivity levels are:[9]

$$x_{ss_{1,2}} = 0.5\left\{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right) \pm \sqrt{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right)^2 + 4\frac{(\alpha - p)(\eta - \rho\beta)}{(\eta - \beta)\delta}}\right\}. \qquad (13)$$

To ensure the existence of interior steady states it is assumed that $|\eta - \beta|\delta$ is sufficiently large so that $|(\alpha - p)\eta / (\eta - \beta)\delta| < 1$. ($\eta \neq \beta$ is implied.)

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[9] It can be shown that

$$x_{ss_{1,2}} = 0.5\left\{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta\hat{y} / c_h^o}\right) \pm \sqrt{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta\hat{y} / c_h^o}\right)^2 + 4\frac{(\alpha - p)(\eta - \rho\beta)}{(\eta - \beta)\delta\hat{y} / c_h^o}}\right\}.$$

The exclusion of $\hat{y}$ and c_h^o from Eq. (13) is due to $\hat{y} = c_h^o$, which is implied the assumptions that there are only two goods, that the budget is instantaneously balanced, and that $\hat{y}$ and c_h^o are, respectively, the income and diet of a perfectly healthy person.

PROPOSITION 4: *If the satisfaction-survival elasticity ratio is smaller than one and equal to the rate of time preference (* $\rho = \beta / \eta < 1$ *), there exists a unique interior steady state with health and productivity being equal to* $1 - [(\alpha - p)\eta / (\eta - \beta)\delta]$. *The greater the survival elasticity (* η *) and the health-depreciating effect of junk-food (* δ *), the better the stationary health and productivity. The greater the composite-diet-generated-satisfaction elasticity (* β *) and the junk and healthy foods' relative taste-price differential (* $\alpha - p$ *), the worse the stationary health and productivity.* (See Appendix A for proof.)

PROPOSITION 5: *If the survival elasticity (* η *) is larger than the eating-satisfaction elasticity (* β *) and* $\rho \neq \beta / \eta$*, there exists a single interior steady state with* $c_{j_{ss}} = c_h^o / \delta$ *and health and productivity level*

$$x_{ss} = 0.5\left\{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right) + \sqrt{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right)^2 + 4\frac{(\alpha - p)(\eta - \rho\beta)}{(\eta - \beta)\delta}}\right\}.$$

It is a center, asymptotically stable spiral, or asymptotically unstable spiral, for people endowed with sufficiently weak time-preferences. It is a saddle point for people endowed with strong time preferences. (See Figure 1a and 1b for illustration and Appendix C and Appendix D for proof.)

Insert Figure 1a here

Insert Figure 1b here

PROPOSITION 6: *If* $\rho\beta < \eta < \beta$, *there exists a single steady state with* $c_{j_{ss}} = c_h^o / \delta$ *and health and productivity level*

$$x_{ss} = 0.5\left\{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right) - \sqrt{\left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right)^2 + 4\frac{(\alpha - p)(\eta - \rho\beta)}{(\eta - \beta)\delta}}\right\}.$$

The larger the rate of time preference the more likely it is that the steady state is a saddle point. (See Figure 2 for illustration and Appendix C and Appendix D for proof.)

Insert Figure 2 here

6. Health and growth-maximizing tax-policy

Governments can increase the personal and aggregate levels of instantaneous and lifetime health and output by taxing junk-food consumption. Consider an economy of N expected-lifetime-utility maximizers with full-capacity incomes equal to their physiologically optimal diets $\hat{y}_1 = c^o_{h1}, \hat{y}_2 = c^o_{h2}, \hat{y}_3 = c^o_{h3}, ..., \hat{y}_N = c^o_{hN}$, with initial health conditions $x_1(0), x_2(0), x_3(0), ..., x_N(0)$ and with relative tastes $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_N$. Suppose that the junk and healthy foods' price ratio, p, in this economy is lower than the junk-healthy relative taste for some, or all, of the members and hence stimulating junk-food consumption. Noting that the rate of change of the *i*-th member's health and income is $\frac{\dot{y}_i(t)}{y_i(t)} = \frac{\dot{x}_i(t)\hat{y}_i}{x_i(t)\hat{y}_i} = \frac{\dot{x}_i(t)}{x_i(t)} = \{1 - \delta[c_{ji}(t)/c^o_{hi}]\}[1 - x_i(t)]$, the following proposition on the aggregate health and growth-maximizing tax-policy for this economy can be made.

PROPOSITION 7: A*n immediately implemented tax rate* $\tau \geq \max\{(\alpha_1 - p), (\alpha_2 - p), (\alpha_3 - p), ..., (\alpha_N - p)\}$ *on junk-food consumption maximizes the aggregate health improvement and facilitates the convergence of the actual aggregate income from* $\sum_{i=1}^{N} x_i(0)\hat{y}_i$ *to the aggregate potential income* $\sum_{i=1}^{N} \hat{y}_i$ *with the highest aggregate production growth rate,* $1 - \sum_{i=}^{N} x_i(t)^2 \hat{y}_i / \sum_{i=}^{N} x_i(t)\hat{y}_i$. (See Appendix A for proof.)

If the agents in this economy have identical skills and employment opportunities and hence identical full-capacity income, the implementation of such a tax rate on junk-food consumption leads to aggregate production growth rates that are equal to the highest aggregate improvement rates of health: $1 - \sum_{i=}^{N} x_i(t)^2 / \sum_{i=}^{N} x_i(t)$. If the agents are identical in every respect - skill, employment opportunities, tastes and initial health - the highest instantaneous growth rates of the aggregate production and health induced by $\tau \geq \alpha - p$ are equal to $1 - x(t)$, where α and x are the common relative taste and level of health.

7. Conclusion

So long that the difference between the relative taste and the relative price of junk-food is positive, the individual's rational diet deviates from the physiologically optimal junk-free diet and generates losses of health, income, longevity and quality of life. The extents of these losses depend on the individual's heath-sensitivity to a physiologically inadequate diet, time-preferences and survival-elasticity. A tax rate that bridges the gap between the relative market price and the highest relative personal taste of junk-food ensures the choice of a junk-free diet by every member of the society. The universal choice of junk-free diet supports the fastest converging path to the highest individual and aggregate levels of health and production. Although tastes are not observed and the costs of their assessment rise with the number of consumers, this tax-policy can be simply implemented by setting the tax rate on junk-food consumption on a very high level. Yet the tax-induced universal abstinence is not Pareto-superior to the free-market outcome that includes cases of partial and total indulgence.

Scare-campaigns are ineffective in the case of rational, sophisticated and hence risk-aware consumers, such as those considered in this paper. Improvements in the preparation, availability, affordability and marketing of healthy food; which reduce the relative taste and increase the relative price of junk food; are the ideal means for diminishing junk-food consumption by rational people.

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Appendix A: Proofs of Propositions 1-4 and 7

Proof of Proposition 1: Recalling Eq. (5), $\alpha < p$ implies that the instantaneous utilities from consuming junk-food are negative. Hence, a maximizer of $\int_0^\infty e^{-\rho t} x(t)^\eta u_t dt$ maintains a junk-free diet $(c_j(t) = 0)$ every *t*. That is, $c_h(t) = y(t) = x(t)\hat{y}$. Recalling Eq. (6), $\lim_{t\to\infty} x = 1$ and, in turn and in recalling Eq. (1), $\lim_{t\to\infty} y = \hat{y}$. Recalling Eq. (2) and Eq. (1), $\lim_{x\to 1} c_h = \hat{y} = c_h^o$. Hence, $\lim_{t\to\infty} c_h = c_h^o$.

Proof of Proposition 2: When $\alpha > p$ and $\rho \to \infty$ the marginal instantaneous satisfactions from the junk-food are positive and as only the present utility matters, the value of future health as well as future consumption are nil and hence $c_h(t) = 0$ and $c_j(t) = x(t)\hat{y}/p$ every instance. Recalling Eq. (6), $\lim_{t\to\infty} x = 0$ and, in turn and in recalling Eq. (1), $\lim_{t\to\infty} y = 0$.

Proof of Proposition 3: The adjoint equation (10) implies, in conjunction with the optimality condition, that along the rational junk-food consumption path the rate of change of the shadow value of health is given by

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -\left[(\eta/\beta)c_j + [1 + (\eta/\beta)\frac{xc_h^o}{\alpha - p}\right](\delta/c_h^o)(1-x) - (1-2x)[1-(\delta/c_h^o)c_j].$$

(A1)

Differentiating this equality with respect to c_j implies that

$$\frac{\partial(\dot{\lambda}/\lambda)}{\partial c_j} \gtreqless 0 \text{ as } x \gtreqless (1+\eta/\beta)/(2+\eta/\beta). \qquad \text{(A2)}$$

Proof of Proposition 4: When $\beta/\eta = \rho$ the second term in the discriminant is equal to zero and hence

$$x_{ss} = 1 - [(\alpha - p)\eta/(\eta - \beta)\delta]. \quad \text{(A3)}$$

If $\beta/\eta > 1$ then $1 - [(\alpha - p)\eta/(\eta - \beta)\delta] > 1$ and hence there is not an interior steady state. The effects of η, β, δ and $\alpha - p$ on x_{ss} are obtained by differentiation.

Proof of Proposition 7: By Proposition 1, an immediately implemented tax rate $\tau > \max\{(\alpha_1 - p), (\alpha_2 - p), (\alpha_3 - p), ..., (\alpha_N - p)\}$ on junk-food ensures that every member i of the society immediately chooses a junk-free diet ($c_{h_i}(t) = y_i(t) = x_i(t)\hat{y}_i$). Recalling equations (6) and (1) and that $\hat{y}_i = c^o_{hi}$, the health-growth rate is, in turn, maximal and the convergence of the actual aggregate product, $Y(t) = \sum_{i=1}^{N} x_i(t)\hat{y}_i$, to the potential aggregate product, $\sum_{i=1}^{N} \hat{y}_i$, is feasible and the fastest:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\sum_{i=1}^{N} \dot{y}_i(t)}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = \frac{\sum_{i=1}^{N} \dot{x}_i(t)\hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = \frac{\sum_{i=1}^{N} x_i(t)[1 - x_i(t)]\hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = 1 - \frac{\sum_{i=1}^{N} x_i(t)^2 \hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i}.$$

Appendix B: Solution of the optimal-control problem and steady states

$$H(t) = \Phi(x)e^{-\rho t}\underbrace{[(\alpha - p)c_j + xc^o_h]}_{Z}{}^{\beta} + \lambda\underbrace{[1 - \delta(c_j/c^o_h)][1 - x(t)]x(t)}_{\dot{x}} \quad \text{(B1)}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -[\Phi_x \underbrace{Z^{\beta}}_{u} - \Phi \underbrace{\beta Z^{\beta - 1} c^o_h}_{u_x}]e^{-\rho t} - \lambda(1 - 2x)[1 - \delta(c_j/c^o_h)] \quad \text{(B2)}$$

$$\frac{\partial H}{\partial c_j} = \Phi e^{-\rho t} \underbrace{\beta Z^{\beta - 1}(\alpha - p)}_{u_{c_j}} - \lambda(\delta/c^o_h)x(1 - x) = 0 \quad \text{(B3)}$$

Eq. (12) is obtained as follows. By differentiating the optimality condition (B3) with respect to time, substituting the right-hand sides of the adjoint equation (B2) and the optimality condition (B3) for $\dot{\lambda}$ and λ:

$$
\begin{aligned}
&-\rho\Phi e^{-\rho t}\beta Z^{\beta-1}(\alpha-p)-(1-\beta)\Phi e^{-\rho t}\beta Z^{\beta-2}(\alpha-p)[(\alpha-p)\dot{c}_j+c_h^o\dot{x}]\\
&+\dot{\Phi}e^{-\rho t}\beta Z^{\beta-1}(\alpha-p)\\
&+\begin{bmatrix}[\Phi_x Z^{\beta}-\Phi\beta Z^{\beta-1}c_h^o]e^{-\rho t}\\ +(1-2x)[1-\delta(c_j/c_h^o)]\dfrac{\Phi e^{-\rho t}\beta Z^{\beta-1}(\alpha-p)}{(1-x)(\delta/c_h^o)x}\end{bmatrix}(1-x)(\delta/c_h^o)x\\
&-\frac{\Phi e^{-\rho t}\beta Z^{\beta-1}(\alpha-p)}{(1-x)x}(1-2x)\dot{x}=0
\end{aligned}
\tag{B4}
$$

Multiplying both sides by $e^{\delta t}/\Phi Z^{\beta-2}\beta(1-\beta)(\alpha-p)$ and collecting terms:

$$
\begin{aligned}
&\left(\frac{\dot{\Phi}}{\Phi}-\rho\right)Z-(1-\beta)[(\alpha-p)\dot{c}_j+c_h^o\dot{x}]\\
&+\left(\frac{\Phi_x}{\Phi}\frac{Z^2}{\beta(\alpha-p)}-\frac{Zc_h^o}{(\alpha-p)}\right)(1-x)(\delta/c_h^o)x\\
&+(1-2x)[1-\delta(c_j/c_h^o)]Z-\frac{(1-2x)}{(1-x)x}Z\dot{x}=0
\end{aligned}
\tag{B5}
$$

Recalling that $\dot{\Phi}/\Phi=\eta(\dot{x}/x)$ and $\Phi_x/\Phi=\eta/x$,

$$
\begin{aligned}
&[\eta(\dot{x}/x)-\rho]Z-(1-\beta)[(\alpha-p)\dot{c}_j+c_h^o\dot{x}]\\
&+\left(\frac{(\eta/x)Z^2}{\beta(\alpha-p)}-\frac{Zc_h^o}{(\alpha-p)}\right)(1-x)(\delta/c_h^o)x\\
&+(1-2x)[1-\delta(c_j/c_h^o)]Z-\frac{(1-2x)}{(1-x)x}Z\dot{x}=0
\end{aligned}
\tag{B6}
$$

By rearranging terms,

$$
\begin{aligned}
&-\rho Z-(1-\beta)(\alpha-p)\dot{c}_j+\left(\frac{\eta}{\beta x}Z-c_h^o\right)(1-x)(\delta/c_h^o)x\frac{1}{(\alpha-p)}Z\\
&+(1-2x)[1-\delta(c_j/c_h^o)]Z-\{[\frac{(1-2x)}{(1-x)x}-\frac{\eta}{x}]Z+(1-\beta)c_h^o\}\dot{x}=0
\end{aligned}
\tag{B7}
$$

Subsequently,

$$\dot{c}_j = -\frac{\rho}{(1-\beta)(\alpha-p)}Z + \left(\frac{\eta}{\beta x}Z - c_h^o\right)(1-x)(\delta / c_h^o)x\frac{1}{(\alpha-p)^2(1-\beta)}Z$$
$$+\frac{1}{(1-\beta)(\alpha-p)}(1-2x)[1-\delta(c_j / c_h^o)]Z$$
$$-\frac{1}{(1-\beta)(\alpha-p)}\{[\frac{(1-2x)}{(1-x)x}-\frac{\eta}{x}]Z+(1-\beta)c_h^o\}\dot{x}$$

(B8)

or, equivalently,

$$\dot{c}_j = -\frac{Z}{(1-\beta)(\alpha-p)}\left\{\begin{array}{l}\rho-(1-2x)[1-\delta(c_j / c_h^o)] \\ -\frac{(\frac{\eta}{\beta x}Z-\gamma\hat{y})(1-x)(\delta / c_h^o)x}{(\alpha-p)} \\ +\{[\frac{(1-2x)}{(1-x)x}-\frac{\eta}{x}]+\frac{(1-\beta)c_h^o}{Z}\}\dot{x}\end{array}\right\} \qquad \text{(B9)}$$

Recalling that $\frac{\dot{x}}{(1-x)x} = 1-\delta(c_j / c_h^o)$ and $Z = (\alpha-p)c_j + xc_h^o$,

$$\dot{c}_j = -\left\{\frac{(\alpha-p)c_j + xc_h^o}{(1-\beta)(\alpha-p)}\right\}\left\{\rho-\frac{(\frac{\eta}{\beta x}Z-c_h^o)(1-x)(\delta / c_h^o)x}{(\alpha-p)}-[\frac{\eta}{x}-\frac{(1-\beta)c_h^o}{Z}]\dot{x}\right\}. \qquad \text{(B10)}$$

Eq. (13) is obtained as follows. The substitution of $\dot{c}_j = \dot{\Phi} = \dot{x} = 0$ into (B6) implies:

$$-\rho Z + \left(\frac{(\eta / x)Z^2}{\beta(\alpha-p)} - \frac{Zc_h^o}{(\alpha-p)}\right)(1-x)(\delta / c_h^o)x + (1-2x)[1-\delta(c_j / c_h^h)]Z = 0$$

.(B11)

By rearranging terms,

$$-\beta(\alpha-p)\rho + (\delta / c_h^o)\eta Z(1-x) - (\delta / c_h^o)\beta\gamma\hat{y}(1-x)x$$
$$+\beta(\alpha-p)(1-2x)[1-(\delta / c_h^o)c_j] = 0 \qquad \text{.(B12)}$$

Recalling that $c_{j_{ss}} = c_h^o / \delta$,

$$-\beta(\alpha - p)\rho + (\delta / c_h^o)\eta[(\alpha - p)/(\delta / c_h^o) + xc_h^o](1-x) \\ - (\delta / c_h^o)\beta c_h^o (1-x)x = 0 \qquad .(B13)$$

Rearranging terms,

$$-\beta(\alpha - p)\rho + [(\alpha - p)\eta + \delta\eta x](1-x) - \delta\beta(1-x)x = 0 \qquad (B14)$$

or, equivalently,

$$(\beta - \eta)\delta x^2 - [(\beta - \eta)\delta + (\alpha - p)\eta]x + (\alpha - p)(\eta - \beta\rho) = 0 \qquad (B15)$$

or, equivalently,

$$x_{ss}{}^2 - \frac{[(\beta - \eta)\delta + (\alpha - p)\eta]}{(\beta - \eta)\delta} x_{ss} + \frac{(\alpha - p)(\eta - \beta\rho)}{(\beta - \eta)\delta} = 0 \qquad (B16)$$

or, equivalently,

$$x_{ss}{}^2 - \left(1 - \frac{(\alpha - p)\eta}{(\eta - \beta)\delta}\right) x_{ss} - \frac{(\alpha - p)(\eta - \rho\beta)}{(\eta - \beta)\delta} = 0 . \qquad (B17)$$

Consequently, the individual's steady-state health level(s) is (are) given by Eq. (13).

Appendix C: Phase-plane diagrams and proof of Propositions 5 and 6

From (6), the isocline $\dot{x} = 0$ is given by a horizontal line in the plane spanned by x and c_j:

$$c_j = c_h^o / \delta . \qquad (C1)$$

From (12) and the definition of Z, along the isocline $\dot{c}_j = 0$

$$\rho - \frac{\{\frac{\eta}{\beta x}[(\alpha - p)c_j + xc_h^o] - c_h^o\}(1-x)(\delta / c_h^o)x}{(\alpha - p)} = 0 \qquad (C2)$$

By rearranging terms the isocline $\dot{c}_j = 0$ is given by:

$$c_j = [\rho\beta / (\delta / c_h^o)\eta] \frac{1}{(1-x)} - \frac{c_h^o[1-\beta/\eta]}{(\alpha - p)} x \tag{C3}$$

The slope of the isocline $\dot{c}_j = 0$ is:

$$\begin{aligned} \frac{dc_j}{dx}\Big|_{\dot{c}_j=0} &= (\rho\beta/\tilde{\delta}\eta) \frac{1}{(1-x)^2} - \frac{c_h^o[1-\beta/\eta]}{(\alpha-p)} \overset{>}{\underset{<}{=}} 0 \\ & as\ (\rho\beta/\tilde{\delta}\eta)(\alpha-p) \overset{>}{\underset{<}{=}} c_h^o[1-\beta/\eta](1-x)^2 \end{aligned} \tag{C4}$$

where $\tilde{\delta} \equiv \delta / c_h^o$.

Recalling that $0 \le x \le 1$ and $\alpha - p > 0$, if $\eta > \beta$ then $\frac{dc_j}{dx}\Big|_{\dot{c}_j=0} \overset{>}{\underset{<}{=}} 0\ as\ (1-x)^2 \overset{<}{\underset{>}{=}} \frac{(\rho\beta/\tilde{\delta}\eta)(\alpha-p)}{c_h^o[1-\beta/\eta]}$. In this case, the isocline $\dot{c}_j = 0$ is U-shaped in the plane spanned by x and c_j. Since in this case,

$$\left(1 - \frac{(\alpha-p)\eta}{(\eta-\beta)\delta}\right) < \sqrt{\left(1 - \frac{(\alpha-p)\eta}{(\eta-\beta)\delta}\right)^2 + 4\frac{(\alpha-p)(\eta-\rho\beta)}{(\eta-\beta)\delta}}$$

$x_{ss_2} < 0$ and only $0 < x_{ss_1} < 1$. Hence, the isocline $\dot{c}_j = 0$ intersects the isocline $\dot{x} = 0$ only once as displayed by Figure 1a and Figure 1b.

If $\eta < \beta$, then $\frac{dc_j}{dx}\Big|_{\dot{c}_j=0} > 0$ and $\frac{d^2c_j}{dx^2}\Big|_{\dot{c}_j=0} < 0$. In this case, the isocline $\dot{c}_j = 0$ is portrayed by an upward-sloped concave curve intersecting the isoclines $\dot{x} = 0$ once, at most. As $1 - \frac{(\alpha-p)\eta}{(\eta-\beta)\delta} > 1$ and the second term in the discriminant of Eq. (13) is negative for $\rho\beta < \eta < \beta$, the stationary health level, in this case, is given by:

$$x_{ss} = 0.5\left\{\left(1-\frac{(\alpha-p)\eta}{(\eta-\beta)\delta}\right)-\sqrt{\left(1-\frac{(\alpha-p)\eta}{(\eta-\beta)\delta}\right)^2+4\frac{(\alpha-p)(\eta-\rho\beta)}{(\eta-\beta)\delta}}\right\}. \tag{C5}$$

Appendix D: Properties of the steady states

To assess the steady states' properties consider the state-transition matrix (Ω) of the linearized form of Eq. (6) and Eq. (12) in the vicinity of steady state:

$$\Omega = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x}(x_{ss}, c_{j_{ss}}) & \frac{\partial \dot{x}}{\partial c_j}(x_{ss}, c_{j_{ss}}) \\ \frac{\partial \dot{c}}{\partial x}(x_{ss}, c_{j_{ss}}) & \frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) \end{bmatrix}. \tag{D1}$$

Since

$$\frac{\partial \dot{x}}{\partial x}(x_{ss}, c_{j_{ss}}) = (1-\tilde{\delta} c_{j_{ss}})(1-2x_{ss}) = (1-\tilde{\delta}(1/\tilde{\delta}))(1-2x_{ss}) = 0 \tag{D2}$$

the eigenvalues of Ω are given by

$$\psi_{1,2} = 0.5\{\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) \pm \sqrt{[\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}})]^2 + 4\frac{\partial \dot{c}}{\partial x}(x_{ss}, c_{j_{ss}})\frac{\partial \dot{x}}{\partial c_j}(x_{ss}, c_{j_{ss}})} \tag{D3}$$

with:

$$\frac{\partial \dot{x}}{\partial c_j}(x_{ss}, c_{j_{ss}}) = -\tilde{\delta}(1-x)x < 0 \tag{D4}$$

$$\begin{aligned}\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) = &-\frac{1}{(1-\beta)}(\rho - R_{2_{ss}}) + R_{1_{ss}}(\eta/\beta)\tilde{\delta}(1-x_{ss}) \\ &- R_{1_{ss}} R_{3_{ss}} \tilde{\delta}(1-x_{ss})x_{ss}\end{aligned} \tag{D5}$$

$$\begin{aligned}\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) = &-\frac{c_h^o}{(1-\beta)(\alpha-p)}(\rho - R_{2\,ss}) \\ &+ R_{1_{ss}}\frac{\{[(\eta/\beta)/(\tilde{\delta}x_{ss})] + [(\eta/\beta)-1]c_h^o\}\tilde{\delta}(1-2x_{ss}) + (\eta c_h^o/\beta)(1-x_{ss})\}}{\alpha - p}\end{aligned} \tag{D6}$$

and where,

$$R_{1_{ss}} = \frac{(\alpha - p)/\tilde{\delta} + x_{ss} c_h^o}{(1-\beta)(\alpha - p)} > 0 \tag{D7}$$

$$R_{2_{ss}} = \frac{(\frac{\eta}{\beta x_{ss}} Z_{ss} - c_h^o)(1 - x_{ss})\tilde{\delta} x_{ss}}{(\alpha - p)} = \left\{ \frac{\eta}{\beta} + \frac{[(\eta/\beta) - 1]\delta}{\alpha - p} x_{ss} \right\} (1 - x_{ss}) \tag{D8}$$

$$R_{3_{ss}} = [\frac{\eta}{x_{ss}} - \frac{(1-\beta)c_h^o}{Z_{ss}}]. \tag{D9}$$

Note that $R_{2_{ss}} \overset{>}{\underset{<}{=}} 0$ as $\frac{\beta}{\eta} \overset{<}{\underset{>}{=}} 1 + \frac{\alpha - p}{\delta x_{ss}}$. When $\eta > \beta$, $\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) > 0$ for people endowed with sufficiently weak time preference (i.e., small ρ). The vertical arrows in Figure 1a display this case. The steady state is a center if $\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) = 0$, asymptotically stable spiral if $\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) < 0$, or asymptotically unstable spiral if $\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) > 0$. (See Eq. (D5).) The latter possibility is displayed in Figure 1a. It is possible, however, that $\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) < 0$ for people with $\eta > \beta$ but with sufficiently strong time preference. This possibility is displayed by the vertical arrows in Figure 1b.

In the case of $\beta > \left(1 + \frac{\alpha - p}{\delta x_{ss}}\right)\eta$, $\rho - R_{2_{ss}} > \rho$ and it is also possible that

$$R_{1_{ss}} \frac{\{[(\eta/\beta)/(\tilde{\delta} x_{ss})] + [(\eta/\beta) - 1]c_h^o\}\tilde{\delta}(1 - 2x_{ss}) + (\eta c_h^o/\beta)(1 - x_{ss})\}}{\alpha - p} < 0. \tag{D10}$$

In this case, the larger the rate of time preference the more likely that $\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) < 0$. The vertical arrows in Figure 2 display this case.

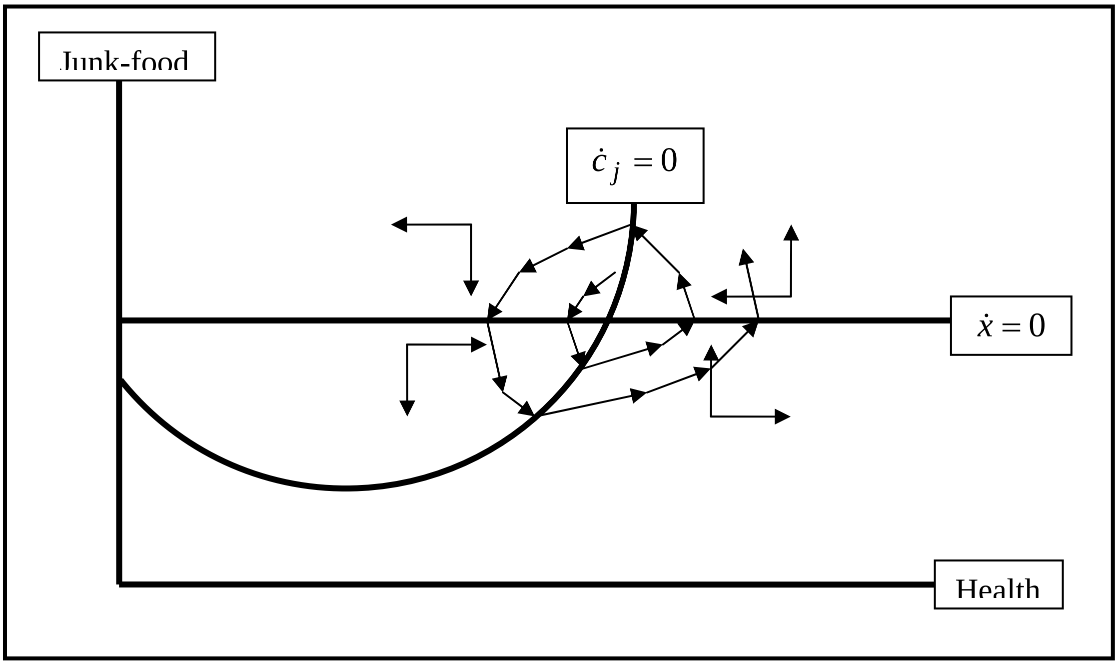

Figure 1a. Phase-plane diagram with $\eta > \beta$ and weak time-preference[10]

[10] The diverging spiral illustrate the case where $\frac{\partial \dot{c}}{\partial c_j}(x_{SS}, c_{j_{ss}}) > 0$. Eq. (D5) reveals that $\frac{\partial \dot{c}}{\partial c_j}(x_{SS}, c_{j_{ss}})$ may alternatively be equal to, or smaller than, zero; which implicates a centre, or a converging spiral, respectively.

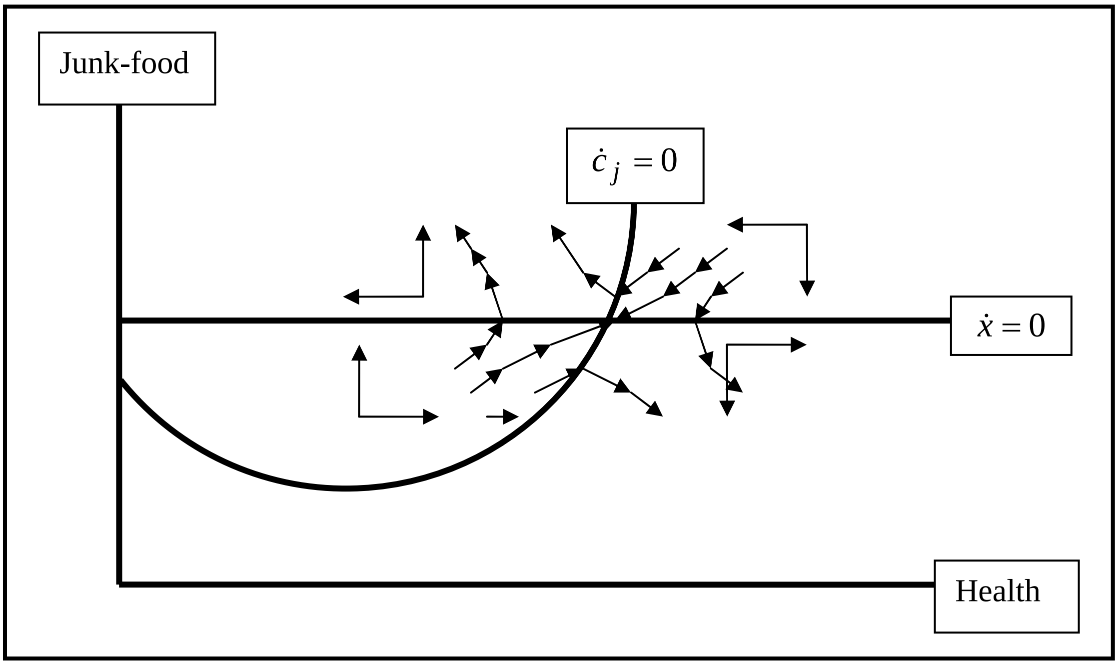

Figure 1b. Phase-plane diagram with $\eta > \beta$ and strong time-preference

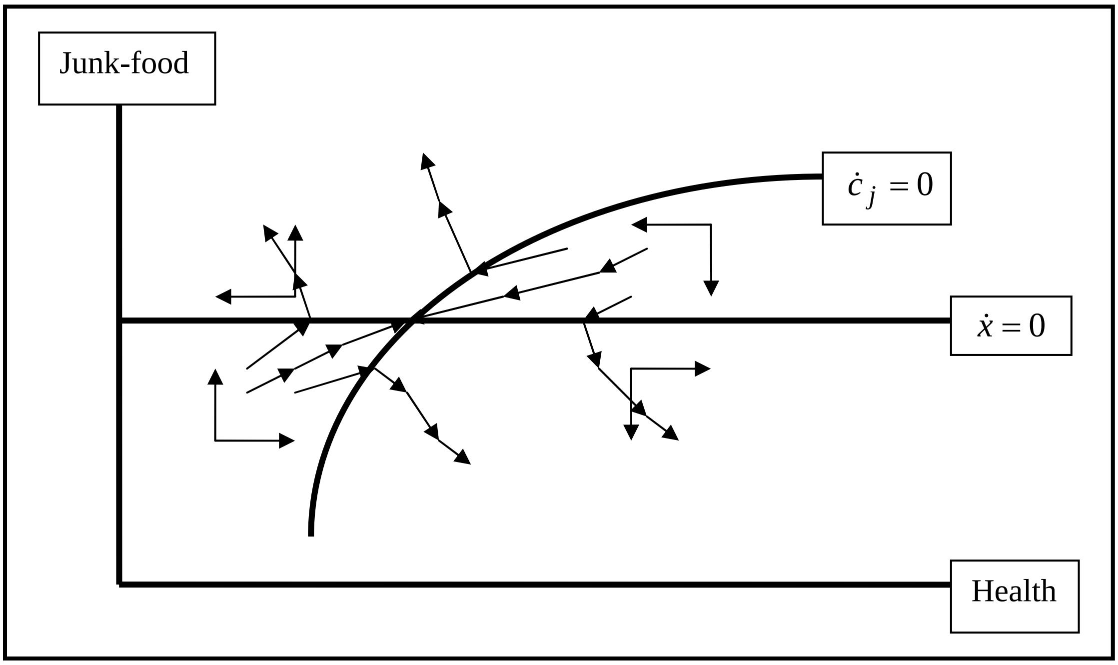

Figure 2. Phase-plane diagram with $\eta < \beta$ and strong time-preference